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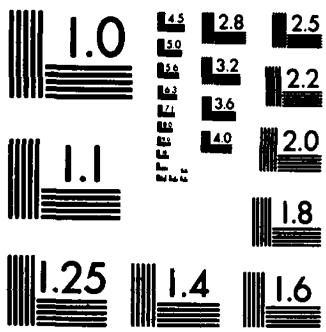

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Interim [redacted] SCIENTIFIC REPORT, GRANT AFOSR-81-0103

1 MAY 1983 TO 30 APRIL 1984

Professor David L. Colton
Department of Mathematical Sciences
University of Delaware
Newark DE 19716

25 APRIL 1984

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MATHEMATICS
OPERATIONS RESEARCH
STATISTICS

April 25, 1984

Dr. Robert N. Buchal
Directorate of Mathematical and
Information Sciences
Air Force Office of Scientific Research
Bolling Air Force Base, DC 20332

Dear Dr. Buchal,

This is my interim report on AFOSR Grant 81-0103 for the period 1 May 1983 to 30 April 1984. Attached are the title pages and abstracts of the papers written during this period. Invited talks on the inverse scattering problem have been given at the Oberwolfach Conference on Scattering Theory, West Germany, and the Technical University of Berlin, West Germany. This summer invited talks will be given on this topic at the Conference on Inverse Problems of Acoustic and Elastic Waves at Cornell University in June, the Conference on the Qualitative Theory of Differential Equations at the University of Alberta in June, and the Conference on the Constructive Methods for the Practical Treatment of Integral Equations at Oberwolfach, West Germany in June.

Sincerely,

David L. Colton

David L. Colton

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0036-1445/84/2601-0000 $01.25/0

THE INVERSE SCATTERING PROBLEM FOR TIME-HARMONIC ACOUSTIC WAVES*

DAVID COLTON†

Abstract. The inverse scattering problem we are considering in this paper is to determine the shape of a sound-soft, bounded, connected obstacle from a knowledge of the time-harmonic incident wave with frequency in the resonant region and the far field pattern of the scattered wave. After some introductory remarks, we begin by describing the method of integral equations and the null-field method for solving the direct scattering problem. This enables us to define an operator **T** mapping the boundary of the scattering obstacle and the incident field onto the far field pattern. The inverse scattering problem is to invert this operator. In order to do this, we first must examine the range of **T**, i.e. to characterize the class of far field patterns, and to establish the existence of $\mathbf{T}^{-1}$ on the range of **T**, i.e. to show the uniqueness of the solution to the inverse scattering problem. This analysis shows that for a given measured far field pattern, in general no solution exists to the inverse scattering problem, and if a solution does exist, it does not depend continuously on the measured data, i.e. the problem is improperly posed. This motivates us to consider a linearized model and to examine various methods *for studying linearized improperly posed problems based on the ideas of a priori* assumptions and compactness arguments. We then consider a simple model problem that focuses on the nonlinear character of the inverse scattering problem and, motivated by our study of the linearized model, reformulate the inverse scattering problem as a problem in constrained optimization. We conclude by considering the numerical solution of this nonlinear optimization problem.

> "Approach your problems from the right end and begin with the answers. Then, one day, perhaps you will find the final question." from "The Hermit Clad in Crane Feathers" in *The Chinese Maze Murders*, by R. Van Gulik.

1. Introduction. This paper is devoted to surveying some of the recent developments associated with the inverse scattering problem for acoustic waves. This problem is perhaps the most famous of the inverse and improperly posed problems arising in mathematical physics; indeed, the name itself is "improperly posed" in the sense that there are numerous "inverse scattering problems" arising in the theory of acoustic wave propagation. Hence it is necessary to clarify at the beginning exactly which of these is to be considered. To get a glimpse of the various types of inverse scattering problems that can arise in applications, it is instructive to first briefly consider some typical problems associated with acoustic scattering. We shall later be more precise concerning the physical derivation and mathematical modeling of both the direct and inverse problem, and hence, for the moment, we shall content ourselves with some simple heuristic descriptions. Consider an a~~c~~coustic wave propagating in a homogeneous, isotropic medium. In the absence of any inhomogeneities, the wave will continue to propagate and nothing of physical interest will happen. However, if there are inhomogeneities present, then the wave will be "scattered" or diffracted" and we can express the total field as the sum of the original "incident" wave and the "scattered" wave. The behavior of the scattered wave will depend on both the incident wave and the nature of the inhomogeneities in the medium, and this in turn is reflected in the mathematical model, e.g. what is the incident wave, is the inhomogeneous region connected, are the boundaries of the inhomogeneous medium bounded or infinite, what type of boundary conditions are appropriate, etc.? The direct problem is, given this information, to find the scattered wave and in particular, its behavior at large distances from the inhomogeneities, i.e. its "far field" behavior. The inverse problem takes this answer to the direct scattering problem as its starting point and asks what is the nature of the inhomogeneities which gave rise to such a far field behavior.

To appear in
SIAM J Math Anal

Dense Sets and Far Field Patterns
in Electromagnetic Wave Propagation

by

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* The research of this author was supported in part by AFOSR Grant 81-0103

Abstract

Is is shown that the electric far field patterns corresponding to the scattering of entire incident fields by a bounded perfectly conducting obstacle are dense in the space of square integrable tangential vector fields defined on the boundary of the unit sphere if and only if there does not exist a Maxwell eigenfunction that is an electromagnetic Herglotz pair, i.e. a solution $\{E,H\}$ of Maxwell's equations defined in all of space such that

$$\lim_{r\to\infty} \frac{1}{r} \iint_{|x|<r} (|E(x)|^2+|H(x)|^2)\,dx < \infty .$$

To appear in
Proceedings of Conference on
Inverse Problems in Acoustic
and Elastic Waves

Far Field Patterns in Acoustic and Electromagnetic Scattering Theory

Abstract

A basic task in the investigation of the inverse scattering problem for time-harmonic acoustic and electromagnetic waves is the study of the class of far field patterns corresponding to the scattering of entire incident fields of a given wave number by a bounded obstacle. Indeed if $\underset{\sim}{T}$ denotes the operator mapping the incident field and scattering obstacle onto the far field pattern, then the inverse scattering problem is to construct $\underset{\sim}{T}^{-1}$ defined on the range of $\underset{\sim}{T}$, and the determination of this range is nothing more than the description of the class of far field patterns. Unfortunately, little is known concerning this class except for the well known fact that the far field patterns are entire functions of their independent (complex) variables for each positive fixed value of the wave number ([3]), i.e. the range of $\underset{\sim}{T}$ is not all of $L^2(\Omega)$ where Ω is the unit sphere. We note that this implies that the inverse scattering problem is an improperly posed problem since the far field patterns are in practice determined from inexact measurements.

Recently Colton ([1]) and Colton and Kirsch ([2]) have investigated the case of acoustic scattering and asked the question if the class of far field patterns corresponding to a fixed scattering obstacle and all entire incident fields is dense in $L^2(\Omega)$. The rather surprising answer to this question is that if the impedance of the scattering obstacle is positive, then the far field patterns are dense in $L^2(\Omega)$, whereas if the scattering obstacle is sound-soft or sound-hard then the far field patterns are dense in $L^2(\Omega)$ if

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Uniqueness Theorems for the Inverse Problem of Acoustic Scattering

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[Received 16 August 1983]

Uniqueness theorems are obtained for the problem of determining the shape of a sound-soft or sound-hard obstacle from a knowledge of (1) the far-field pattern at a fixed value of the wave number and a finite number of distinct incident fields, or (2) the total scattering cross section for an interval of wave numbers and the incident field propagating in an arbitrary direction.

1. Introduction

THE INVERSE SCATTERING PROBLEM for acoustic waves forms the basis of a wide variety of areas in the engineering sciences involving remote sensing and imaging, and for this reason has been the object of intensive study by scientists in a number of diverse disciplines. Since around 1970 progress has been particularly rapid, and for a survey of these recent results we refer the reader to the expository papers by Colton (1983) and Sleeman (1982). However, in this intensive and prolonged effort there are at present only a small number of results available on the uniqueness of the solution to the inverse scattering problem. The purpose of this paper is to add several additional uniqueness theorems to this sparse collection, our motivation coming from some recent numerical results of Andreas Kirsch who considers inverse scattering problems not covered by previously known uniqueness theorems and which in fact seem to exhibit non-uniqueness (Kirsch, 1982). However, before we can describe our results we must be more precise as to what inverse scattering problem we are considering, since the term "inverse scattering problem" is not uniquely defined. To this end we consider a plane time-harmonic acoustic wave moving in the direction $\boldsymbol{\alpha}$ that is scattered by a bounded connected domain D in $\mathbb{R}^3$ which is assumed to be either "sound-soft" or "sound-hard". Then if we factor out the periodic dependence on time and denote the total field by $u(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^3 \setminus D$, the scattered field by $u^s(\mathbf{x})$, and the (positive) wave number by k, we have that

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Proceedings of Strathclyde
Conference on Scattering Theory

Uniqueness of Solutions to the Inverse Acoustic Scattering Problem

David Colton and B D Sleeman

1. INTRODUCTION

The inverse acoustic scattering problem forms the basis of a wide range of problems in the engineering sciences including for example remote sensing and imaging and consequently has been the object of intensive study in recent years. For an overview of recent contributions we cite the expository articles [1] and [6]. Despite this intensive research there are only a few results concerned with the question of uniqueness of solutions to the inverse scattering problem. In this paper we review known uniqueness results and report on some new developments. Full proofs of the new results are to be found in [3].

Before we can adequately describe what is meant by the inverse scattering problem it is necessary to recall some fundamental notions concerning the direct problem. To this end we consider a plane time-harmonic acoustic wave moving in the direction $\underset{\sim}{\alpha}$ that is scattered by a bounded connected domain (the scattering obstacle) D in $\mathbb{R}^3$ assumed to be either 'sound-soft' or 'sound-hard'. If we suppress the assumed harmonic time dependence and denote the total field by $u(\underset{\sim}{x})$, $\underset{\sim}{x} \in \mathbb{R}^3 \backslash D$, the scattered field by $u^s(\underset{\sim}{x})$ and the (positive) wave number by k then $u(\underset{\sim}{x}) = e^{ik\underset{\sim}{x}.\underset{\sim}{\alpha}} + u^s(\underset{\sim}{x})$, $|\underset{\sim}{\alpha}| = 1$, must satisfy the following boundary value problem for the Helmholtz equation

$$\Delta_3 u + k^2 u = 0 \quad \text{in} \quad \mathbb{R}^3 \setminus \bar{D} \tag{1.1}$$

$$u = 0 \quad \text{on} \quad \partial D \, , \tag{1.2a}$$

or

$$\frac{\partial u}{\partial \underset{\sim}{\nu}} = 0 \quad \text{on} \quad \partial D \, , \tag{1.2b}$$

To appear in
Proc. Edinburgh Math Soc

The Strong Maximum Principle for
*the Heat Equation**

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February, 1984

**This research was supported in part by AFOSR Grant 81-0103.*

Abstract: A mean value theorem is derived for the heat equation and is used to prove the strong maximum principle for solutions of the heat equation.

The strong maximum principle for harmonic functions is usually arrived at by appealing to the mean value theorem (c.f. [1], p. 53). It is also of course possible to simply appeal to the Hopf maximum principle ([2]), but using sledge hammers to kill flies is generally viewed as aesthetically unpleasing. In contrast to the case of harmonic functions, the only proof of the strong maximum principle for the heat equation that is known to me is to invoke Nirenberg's strong maximum principle for parabolic equations ([2]). As in the case of harmonic functions, it seems desirable to provide a direct proof of this result without having to go through the subtle comparison arguments that are employed in the more general case. The purpose of this note is to provide a proof of the strong maximum principle for the heat equation based on a mean value theorem for solutions of the heat equation which we derive below. Such an approach provides a straightforward and simple proof of the strong maximum principle which avoids most of the detailed estimates of the proof of the maximum principle for more general parabolic equations.

For the sake of simplicity we shall only consider the case of the heat equation in three space dimensions.

Theorem: Let $u(\underset{\sim}{x},t)$, $\underset{\sim}{x} \in \mathbb{R}^3$, satisfy the heat equation

$$\Delta_3 u = u_t$$

To appear in
Complex Variables. Theory and Application

ANALYTIC SOLUTIONS OF THE HEAT EQUATION AND SOME FORMULAS FOR LAGUERRE AND HERMITE POLYNOMIALS*

by

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*The research of the first author was supported in part by AFOSR Grant 81-[illegible]3 and the second author was supported in part by NSF Grant MCS-8301842.

Abstract

We consider analytic solutions of the heat equation $u_{xx}+u_{yy} = u_t$ defined in a cylinder and show that any such solution can be expanded in a series of polynomial solutions to the heat equation. If we define the independent complex variables z and $\bar{z}$ by $z = x+iy$, $\bar{z} = x-iy$, where x and y are independent complex variables, it is shown that any real-valued analytic solution of the heat equation is uniquely determined by its values on $\bar{z}=0$ or $t=0$. Using this result, and expressing the above mentioned polynomial solutions to the heat equation in terms of Laguerre polynomials, we obtain some generating functions for Laguerre polynomials, as well as connection formulas between products of Hermite polynomials and Laguerre polynomials of argument $r^2 = x^2+y^2$. These connection formulas generalize a well known result of Feldheim.

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